

## Analysis of Torsional Vibrations of Rotary Winding Machines

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### ABSTRACT

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This work is chosen to analyze the rotary winding machines because these machines are existed in our country and this analysis is practical from the side of input values and getting the results. Good rotary winding machine design practice demands the analysis of the system to insure that the reaction due to operation of the machine will not cause damaging vibrations. The widest phenomenon in vibration forms in mechanical transmission is the torsional vibration. The aim of this work is the analysis of the torsional vibrations of these rotary winding machines and to minimize that vibrations and to alleviate recurring costly maintenance problems.

To minimize system dynamic torques it is necessary to spread the torsional natural frequencies. This is best achieved by lowering the fundamental torsional natural frequency. In main rotary winding machine drives this is most readily accomplished by lowering the torsional stiffness at the lead spindle location or, by making the lead spindle torsional flexibility greater by increasing the shaft separation or, if necessary, by using a torsionally flexible spacer. Torsional stiffness is inversely proportional to shaft length. For the solution of the problem one must first of all estimate the physical system parameters taking into consideration the real set of components of a rotary winding machine and then to transform them into a mathematical model. In this work there was also the method of calculation model presented.

The equations of motion of the whole vibrating system are defined and the study include solution of torsional vibration of this machine by using Jacobi method as method of solution because it is easy to program and accurate. A computer program (Math Lab) has been used in order to Facilitate the solution because it is contain already comments specialized to solve the vibration problems. Finally, finding the final results gave a good vision to forecast the failure of the machine that could caused either by errors in design or operating conditions.

Key words : Torsional vibration, Torsional frequency, Torsional torque, Rotary machines, Torsional drives, Torque amplification factors, Dynamics of winding drives, Jacobi method.

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## **1. INTRODUCTION**

Vibration of a physical structure often is thought of in terms of model consisting of a mass and a spring [1]. The vibration of such a model, or system, may be free or forced. In free vibration, there is no energy added to the system but rather the vibration is the continuing result of an initial disturbance. In a real system, energy dissipation causes the amplitude of free vibration to decay continuously to a negligible value. Such free vibration sometimes is referred to as transient vibration [2]. Forced vibration continues under steady-state conditions because energy is supplied to the system continuously to compensate for that dissipated in the system. The forcing frequency at which energy is supplied appears in the vibration of the system. The vibration of the system depends upon the relation of the forcing frequency to the natural frequency. This relationship is a prominent feature of the analytical aspects of vibration [2].

The technology of vibration embodies both theoretical and experimental facets prominently. Thus, methods of analysis and instruments for the measurement of vibration are of primary significance. The results of analysis and measurement are used to evaluate vibration environments, to devise testing procedures, and testing machines, and to design and operate equipment and machinery [3]. In this work the objective is to eliminate vibrations or reduce their severity or, alternatively, to design equipment to withstand their influence. The wide phenomenon in vibration arms in mechanical transmission is the torsional vibration. Therefore the theory torsional vibration and its applications have reached large expansion of this branch was due to practice needs [4].

Torsional vibration involves angular oscillations of the rotors of a machine. Dynamics problems associated with rotary machines drive system generally result from the torsional vibrations. The torsional vibration problem arises when the natural frequencies of a system and/ or its components are within the operating range of the system, critical speeds may exist in which dynamic effects are predominant and give rise to large amplitude vibration. These vibrations can have a detrimental effect on fatigue life, regulating system performance, product quality and noise levels. For large rotating machinery the mechanical system consists of several rotors that are connected by relatively shafts and couplings. For example, Figure (1) is the photograph of the drum of winding machine.

It has the large diameter rotor bodies section and relatively flexible shafts extensions. Each rotor in the system has oscillated following a torsional disturbance to the machine about its rotational axis, resulting in twisting in the shafts and to a lesser extent in the large diameter rotor bodies themselves. For some machines involving geared rotor connections, for example, there are many several rotor axes of rotation. The twisting oscillations following several torsional disturbances to the machine may be sufficient to cause fatigue damage to the shafts of the machine and the other components [2].

In the design of rotating machinery, torsional vibration analysis is vital for ensuring reliable machine operation. If shaft and rotating component failures occur on these large machine as a result of shaft torsional oscillations, the consequences can be catastrophic. In the worst case, the entire machine can be wrecked as a result of the large unbalancing forces that can arise following shaft separation and turbine blade failures, and this has actually occurred. There is also potential for loss of human life, for these reasons great attention is generally taken at the design stage to ensure that high-speed rotating machines have the required torsional capability.



Figure (1): Photograph of Drum of Winding Machine [5].

## 2. EXPLANATION OF THE SYSTEM

### 2.1 Defining the problem

The equipment arrangement of a modern winding machine is shown in Figure (1) consisting of several stage of gear boxes in order to control speed either reduction or magnification, because level of speed is controlling the wind quality of the cable. Also, there are large pulleys in order to organize the tension and reduce the diameter of the cable as final process. From a mechanical point of view, the determination and correction of responses of the system began with an evaluation of each individual stand considered as a separate entity. As the operation proceeds through each of these stands, at different speeds, each stand in turn is subjected to the initial different speed operation. At the instant of operation, each of these stands can be considered an integral system, consisting of all rotating components-rolls, pulleys, gears, motor rotors and their interconnecting shafts, couplings and spindles. With due regard for the composition, size, shape, dimension, and strength of all parts, the task at hand is to determine the response of each of these mechanical systems to the load imposed at the instant of operation.

### 2.2 Physical analysis of the system

To reduce the problem to a manageable form, many of these contributing factors must be assumed to have minor influence on the character of the behavior of the drive train. Fortunately, there is an abundance of technical literature dealing with the construction of a suitable physical model [6, 7, 8].

Most of this development is devoted to the consideration of steady- state vibration, the usual concern in reciprocating engine and compressor drives, but is equally applicable to the investigation of transient torsional vibration. This phase of the analysis is, consequently, clear-cut and can be relied upon to yield precise information about the natural frequencies of the system, the stresses which will occur as a result of torsional deflection at various stations and to suggest possible changes which can be made to reduce the torque



stages and connected by means of pin bushes ( flexible couplings) which are used elastic sleeve pin couplings instead of attachment bolts. Through the sleeve pin is usually oil resistant rubber. Also this winding machine has braking system, the system has contained two wheel drums, each one has capacity to be loaded with eight cable pulleys [5].

### 2.3 Mechanical analysis of the system :

To adapt the physical drive system shown in Figure ( 2 ) to a system may be validly modeled as a series of concentrated inertias connected by massless torsional springs and dampers. A typical block diagram model of a rotary winding machine is illustrated in Figure ( 3 a ). In order to make an analysis the complete rotary winding machine drive shown in Figure ( 2 ) is reduced to an equivalent spring mass system as shown in Figure ( 3 b).

The drive system is transformed into a single line spring mass system by the application of fundamental equations of mechanics, Some of which are given as :

a ) for ease of analyzing the motor parts, choose the motor shaft speed as the base speed and designate the other shaft speeds as ((n)) where ((n)) equals the speed ratio of the other shafts with respect to the base .

b ) multiply all springs and inertias by  $n^2$  . The effective stiffnesses of the shafts or couplings on the high speed side of a gear box, referred to the low speed is the actual stiffness multiplied by the gear ratio squared.

$$1. \quad K = \frac{G J_p}{L}, \quad K \text{ is the torsional stiffness of the shaft in N.m / rad}$$

- G is the modulus of rigidity of the shaft in  $N / m^2$   
(  $G = 75 \times 10^9 \text{ N / m}^2$  , for steel structure A - 36 ) [5].
- L is the length of the shaft in m.
- $J_p = \frac{\pi D_s^4}{32}$

$J_p$  is the polar area moment of inertia of the shaft in  $m^4$  .

$D_s$  is the diameter of the shaft in m (  $D_s = 0.075 \text{ m}$  , for all shafts ) [5].

$$2. \quad y = \frac{P}{K_t} = \frac{P \ell^3}{3 E J}$$

- y is the bending deflection of the gear teeth measured on the pitch circle in m.  
( no slippage, i.e., no shear deflection ).
- P is the circumferential force on the gear teeth in N.
- $K_t = \frac{3 E J}{\ell^3}$
- $K_t$  is the Stiffness of gear tooth in  $N / m$ .
- E is the modulus of elasticity of the deflected tooth in  $N / m^2$ .  
(  $E = 210 \times 10^9 \text{ N / m}^2$  , for steel ) [5].
- $\ell$  is length of the deflected tooth in m.

$$\bullet \quad J = \frac{\pi D_g^4}{64}$$

- J is the diametral area moment of inertia of gear in  $m^4$ .
- $D_g$  is the pitch circle diameter of the gear in m.

$$3. \quad K_{eq} = \frac{G J_p}{L_{eq}} = \frac{r_1^2}{\frac{1}{K_{t1}} + \frac{1}{K_{t2}}}$$

- $K_{eq}$  is the equivalent rigidity of the transmission teeth of gears in  $(N.m) / (rad)$ .  
(  $G = 80 \times 10^9 \text{ N} / m^2$ , for steel of gears )[5].
- $r_1$  is the pitch circle radius of the first driver gear in m.
- $K_{t1}$  is the stiffness of first meshed driver gear in  $N / m$ .
- $K_{t2}$  is the stiffness of Second meshed driven gear in  $N / m$ .

$$\bullet \quad L_{eq} = \frac{G J_p}{K_{eq}} = \frac{G J_p \left( \frac{1}{K_{t1}} + \frac{1}{K_{t2}} \right)}{r_1^2}$$

- $L_{eq}$  is the reduced length of an equivalent shaft in m.

$$4. \quad I = m K_o^2 = m \frac{R^2}{2}$$

- I is the polar mass moment of inertia in  $kg.m^2$ .
- m is the mass of the rotating part in kg.
- $K_o^2 = \frac{R^2}{2}$
- $K_o$  is the radius of gyration of the rotating part in m.
- R is the radius of the rotating part in m.

$$5. \quad C = 2 \xi \sqrt{K I} = 2 \xi \omega_n I$$

- C is the viscous damping coefficient in  $(N.m) / (rad/s)$ .

$$\bullet \quad \xi = \frac{C}{C_c}$$

- $\xi$  is the damping ratio factor in dimensionless.

(  $\xi = 0.8$  for the designed winding machine)

$$\bullet \quad \omega_n^2 = \frac{K}{I}$$

- $\omega_n$  is the natural frequency of the rotating part in  $rad / s$ .

$$\bullet \quad C_c = 2 \sqrt{K I} = 2 \omega_n I$$

- $C_c$  is the critical viscous damping coefficient in (N.m) / (rad/s).
6. Two springs in series  $K_1$  and  $K_2$  can be represented by an equivalent spring.

$$K = \frac{1}{\frac{1}{K_1} + \frac{1}{K_2}}$$

7. Two springs in parallel  $K_1$  and  $K_2$  can be represented by an equivalent spring  $K$ .

$$K = K_1 + K_2$$

8. Rotational spring constant  $K$  of a hollow circular shaft of outside diameter  $D_o$ , inside diameter  $D_i$  and length  $L$ .

$$K = \frac{G J_{op}}{L} - \frac{G J_{ip}}{L} = \frac{G \pi (D_o^4 - D_i^4)}{32 L}$$

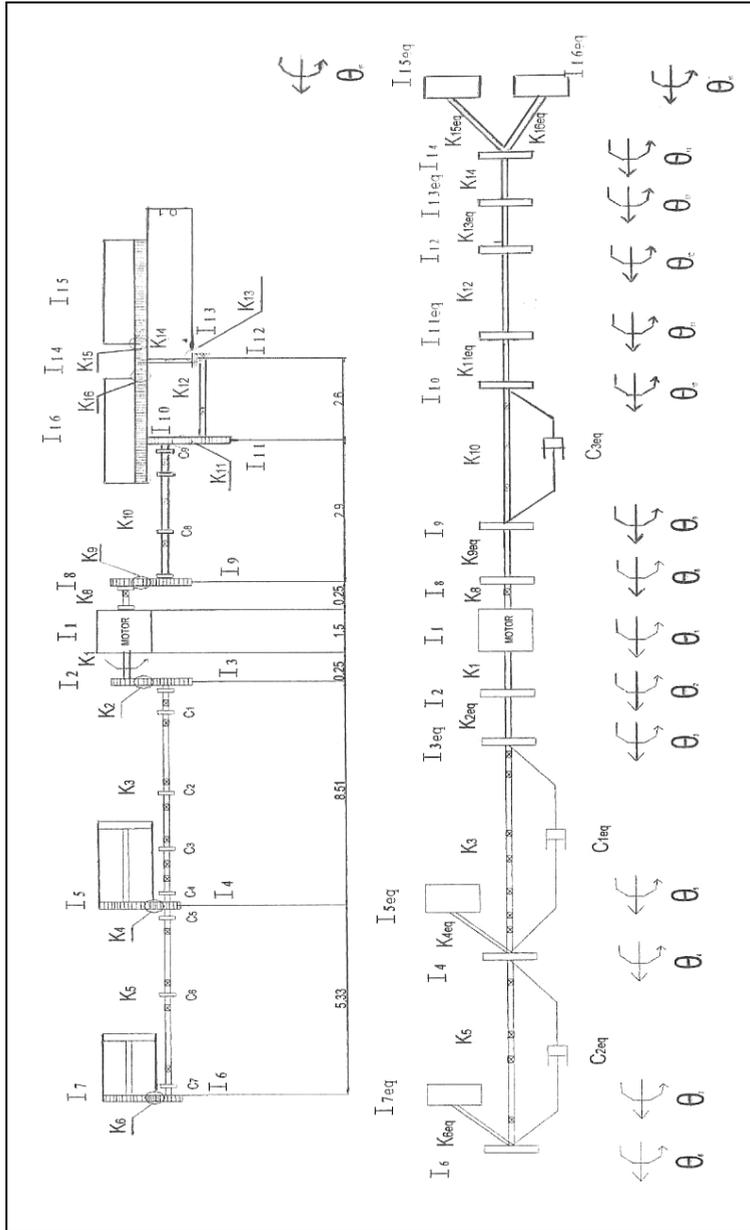


Figure (3): An Adapted Block Diagram Model of a Rotary Winding Machine.

### 2.4 Mathematical Analysis of The System

From point of view of mathematic analysis of general torsion system of the rotary winding machine system there is but no difference between subsidiary flexible bindings and the

terms that describe so called branching of the system, example due to influence of gear-insertion.

The derivation of equations of motion of the mathematical idealized model of rotary winding machine system shown in Figure (3 b) can be obtained by applying Lagrange's equations or by applying Newton's second Law. Thus the equations of motion of the winding machines system shown in Figure (3 b) are :

$$\begin{aligned}
 I_1 \ddot{\theta}_1 + K_1 (\theta_1 - \theta_2) + K_8 (\theta_1 - \theta_8) &= + T_M \\
 I_2 \ddot{\theta}_2 + K_1 (\theta_2 - \theta_1) + K_{2eq} (\theta_2 - \theta_3) &= 0 \\
 I_3 \ddot{\theta}_3 + C_{1eq} (\dot{\theta}_3 - \dot{\theta}_4) + K_{2eq} (\theta_3 - \theta_2) + K_3 (\theta_3 - \theta_4) &= 0 \\
 I_4 \ddot{\theta}_4 + C_{1eq} (\dot{\theta}_4 - \dot{\theta}_3) + C_{2eq} (\dot{\theta}_4 - \dot{\theta}_6) + K_3 (\theta_4 - \theta_3) + K_{4eq} (\theta_4 - \theta_5) + K_5 (\theta_4 - \theta_6) &= 0 \\
 I_5 \ddot{\theta}_5 + K_{4eq} (\theta_5 - \theta_4) &= 0 \\
 I_6 \ddot{\theta}_6 + C_{2eq} (\dot{\theta}_6 - \dot{\theta}_4) + K_5 (\theta_6 - \theta_4) + K_{6eq} (\theta_6 - \theta_7) &= 0 \\
 I_7 \ddot{\theta}_7 + K_{6eq} (\theta_7 - \theta_6) &= 0 \\
 I_8 \ddot{\theta}_8 + K_8 (\theta_8 - \theta_1) + K_{9eq} (\theta_8 - \theta_9) &= 0 \\
 I_9 \ddot{\theta}_9 + C_{3eq} (\dot{\theta}_9 - \dot{\theta}_{10}) + K_{9eq} (\theta_9 - \theta_8) + K_{10} (\theta_9 - \theta_{10}) &= 0 \\
 I_{10} \ddot{\theta}_{10} + C_{3eq} (\dot{\theta}_{10} - \dot{\theta}_9) + K_{10} (\theta_{10} - \theta_9) + K_{11eq} (\theta_{10} - \theta_{11}) &= 0 \\
 I_{11} \ddot{\theta}_{11} + K_{11eq} (\theta_{11} - \theta_{10}) + K_{12} (\theta_{11} - \theta_{12}) &= 0 \\
 I_{12} \ddot{\theta}_{12} + K_{12} (\theta_{12} - \theta_{11}) + K_{13eq} (\theta_{12} - \theta_{13}) &= 0 \\
 I_{13} \ddot{\theta}_{13} + K_{13eq} (\theta_{13} - \theta_{12}) + K_{14} (\theta_{13} - \theta_{14}) &= 0 \\
 I_{14} \ddot{\theta}_{14} + K_{14} (\theta_{14} - \theta_{13}) + K_{15eq} (\theta_{14} - \theta_{15}) + K_{16eq} (\theta_{14} - \theta_{16}) &= 0 \\
 I_{15} \ddot{\theta}_{15} + K_{15eq} (\theta_{15} - \theta_{14}) &= -T_{01} \\
 I_{16} \ddot{\theta}_{16} + K_{16eq} (\theta_{16} - \theta_{14}) &= -T_{02} \dots\dots\dots (1)
 \end{aligned}$$

The above differential equations second orders with constants coefficients can be written in matrix form as follows :-

$$M\ddot{\theta} + C\dot{\theta} + K\theta = F(t) \dots\dots(2)$$

Where,  
M is a diagonal matrix mass moments of inertia.  
C is a symmetric damping matrix.  
K is a symmetric stiffness matrix.

$\theta$  is a vector angular displacements of the masses.

$\dot{\theta}$  is a vector angular velocities of the masses.

$\ddot{\theta}$  is a vector angular accelerations of the masses.

F(t) is a vector excited torques.

### 3. SOLUTION OF THE PROBLEM

For low speed motors the problem of torsional vibration of a shafting system is usually ignored, because the torsional natural frequencies of a shafting system are much higher than its operating speed so that their effects can be ignored. However, for high speed motors, their effects cannot be ignored and have to be completely studied; and our problem on the rotating winding machine one of this type. Such problems supply calculation mathematical models with many degrees of freedom as shown in Equation (1) that can be solved using numerical techniques and computers and there are many suitable methods to solve the problem such as Holzer method and Jacobi method [10], whereas Holzer method is in fact a systematic tabulation of the frequency equation of the vibratory system, method of determining the shapes and frequencies of torsional modes of vibration of a system and in this work used Jacobi method because this method is an algorithm for determining the solution of the system of linear equation with largest absolute value in each row column dominated by the diagonal element. Otherwise it is method of solving matrix equation on a matrix that zeros along its main diagonal. Advantages of the Jacobi method are easy to program and accurate; so that this method is explained in the following article.

Jacobi Method: The free vibration equation for an undamped system is obtained from the general Equation (2), when F(t) and C are absent.

Therefore :

$$M\ddot{\theta} + K\theta = 0 \dots\dots (3)$$

Consider the real eigen value problem with given symmetric matrices K and M with M positive definite, hence the problem is to determine the eigen vectors  $v_r$  and the eigen values  $\omega_r^2$  ( $r = 1, \dots, n$ ) which satisfy

$$K v = \omega^2 M v \dots\dots\dots (4)$$

Where (n) are the real roots for  $\omega^2$ . If K is singular, at least one root is zero. If K is positive definite all roots are positive. The (n) roots determine the (n) natural frequencies of the system. When a natural frequency  $\omega_r$  is known, it is possible to return to equation (4) and solve for the corresponding vector  $v_r$  to within a multiplicative constant.

There are (n) independent vectors  $v_r$  ( $r = 1, \dots, n$ ) corresponding to the (n) natural frequencies  $\omega_r$  ( $r = 1, \dots, n$ ) which are known as eigen values. The complete solution to the eigen value problem of Equation (4) consists of (n) eigen values and (n) corresponding eigen vectors. These can be assembled compactly into matrices. Let the eigen vectors  $v_r$

corresponding to the eigen value  $\omega_r^2$  have elements  $U_{jr}$  (the first subscript indicates which row, the second subscript indicates which eigen vector ). The (n) eigenvectors then can be displayed in single square matrix V, each column of which is an eigenvector.

$$V = [U_{jr}] \dots\dots (5)$$

Where  $j = 1, \dots, n$  and  $r = 1, \dots, n$

The matrix V is called the modal matrix for the eigen value problem, equation (4).

The (n) eigen values  $\omega_r^2$  can be assembled into a diagonal matrix  $\Omega^2$  which is known as the spectral matrix of the eigen value problem, Equation (4).

$$\Omega^2 = [\omega_r^2] \dots\dots (6)$$

By using the modal and spectral matrices it is possible to assemble all of these relations into a single matrix equation :

$$K V = M V \Omega^2 \dots\dots (7)$$

Equation (7) provides a compact display of the complete solution of the eigen value problem, Equation (4). By premultiplication the both sides of Equation (7) by  $V^T$  and after arrangement, it has reduced to :

$$\frac{V^T K V}{V^T M V} = \Omega^2 \dots\dots (8)$$

The problem of Equation (7) is reduced to an eigen value problem for a simple symmetric matrix  $A = M^{-1/2} K M^{-1/2}$  with modal matrix  $U = M^{1/2} V$  as follows :-

$$\begin{aligned} K V &= M V \Omega^2 \\ K M^{-1/2} M^{1/2} V &= M^{1/2} M^{1/2} V \Omega^2 \\ (M^{-1/2} K M^{-1/2}) (M^{1/2} V) &= (M^{1/2} V) \Omega^2 \\ A U &= U \Omega^2 \dots\dots (9) \end{aligned}$$

The solution of this eigen value problem, Equation (9) provides the spectral matrix  $\Omega^2$  of the original problem. The modal matrix U has the property that its columns are normalized so that :

$$U_r^T U_r = 1, \text{ i.e., } U^T = U^{-1} \text{ and } U^{-T} = U$$

By post multiplication the both sides of the Equation (9) by  $U^T$ , it has reduced to :

$$A U U^T = U \Omega^2 U^T$$

$$A U U^{-1} = U \Omega^2 U^T$$

$$A = A I = U \Omega^2 U^T \dots\dots (10)$$

Where, I is identity matrix or a diagonal unit matrix.

The basic computational operation in this method is the resolution of a single symmetric matrix A into its modal matrix U and its spectral matrix  $\Omega^2$  according to the relations shown in Equation (10). Where  $\Omega^2$  is a diagonal matrix of the eigen values  $\omega_r^2$ , and the columns U are the eigen vectors  $U_r$  for the eigen value problem shown in Equation (9) .

Also, we can say that by premultiplication the both sides of Equation (9) by  $U^T$ , it has reduced to :

$$U^T A U = U^T U \Omega^2$$

$$U^T A U = U^{-1} U \Omega^2$$

$$U^T A U = I \Omega^2 = \Omega^2 \dots\dots (11)$$

In comparison Equation (11) to Equation (8) we have deduced that :

$$V^T M V = I \quad \text{and} \quad V^T K V = \Omega^2$$

To obtain the modal matrix V of the original problem in Equation (7), it is necessary to perform the matrix multiplication :

$$V = M^{-1/2} U \dots\dots (12)$$

which follows from inverting the definition of U. It remains to indicate how the resolution of Equation (10) is obtained by successive rotations.

#### 4. RESULTS OF THE SOLUTION

By substitution all the values of the parts of the system and by solved it numerically by Jacobi method which is programmed by Math-Lab program we get the results for determining the frequencies, eigenvectors, and plotting the eigenvectors against the distances of the system [9].

##### A ) The input data :

- a ) The constants data of the system, the mass moments of inertia matrix, the damping coefficients matrix and the stiffness coefficients matrix.
- b ) The size of square matrix  $A = M^{-1/2} = I^{-1/2}$   
Where M = I is the mass moments of inertia matrix.
- c ) The size of square matrix  $C = A * K$  .  
Where K is the stiffness matrix .
- d ) The size of square matrix  $D = C * A$  .
- e ) The distances between the masses .

A =

Columns 1 through 15

Column 16

0.2500	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	5.1920	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	1.2970	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	1.2970	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0.0210	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	1.2970	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0.0160	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	5.1920	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	1.2970	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	5.1920	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	1.2970	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	7.2550	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	7.2550	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0.3600	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0.0180	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0.0180	0.0180

C =

1.0e+008 \*

Columns 1 through 15

Column 16

0.0047	-0.0023	0	0	0	0	0	-0.0023	0	0	0	0	0	0	0	0	0
-0.0484	0.6048	-0.5564	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	-0.1390	0.1393	-0.0004	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	-0.0004	2.2248	-2.2238	-0.0006	0	0	0	0	0	0	0	0	0	0	0
0	0	0	-0.0360	0.0360	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	-0.0006	0	6.4353	-6.4347	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	-0.0794	0.0794	0	0	0	0	0	0	0	0	0	0
-0.0484	0	0	0	0	0	0	0.9317	-0.8833	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	-0.2207	0.2217	0.0010	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0.0042	0.8875	-0.8833	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	-0.2207	0.2218	-0.0012	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	-0.0065	1.8419	-1.8354	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	-1.8354	2.0052	-0.1690	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	-0.0084	0.9536	-0.4726	-0.4726	-0.4726
0	0	0	0	0	0	0	0	0	0	0	0	0	0	-0.0236	0.0236	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	-0.0236	0	0.0236

D =

1.0e+009 \*

Columns 1 through 15

Column 16

0.0001	-0.0012	0	0	0	0	0	-0.0012	0	0	0	0	0	0	0	0	0
-0.0012	0.3140	-0.0722	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	-0.0722	0.0181	-0.0000	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	-0.0000	0.2886	-0.0047	-0.0001	0	0	0	0	0	0	0	0	0	0	0
0	0	0	-0.0047	0.0001	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	-0.0001	0	0.8347	-0.0103	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	-0.0103	0.0001	0	0	0	0	0	0	0	0	0	0
-0.0012	0	0	0	0	0	0	0.4837	-0.1146	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	-0.1146	0.0288	0.0005	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0.0005	0.4608	-0.1146	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	-0.1146	0.0288	-0.0008	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	-0.0008	1.3363	-1.3316	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	-1.3316	1.4548	-0.0061	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	-0.0061	0.0343	-0.0009	-0.0009	-0.0009
0	0	0	0	0	0	0	0	0	0	0	0	0	-0.0009	0.0000	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	-0.0009	0	0.0000

**B ) The output data :**

- 1 ) The output data are :-
  - a ) The eigen values squared (Lam) ( natural frequency squared ) in ( rad/s )<sup>2</sup>.
  - b ) The eigen vectors ( **V** ), ( mode shapes ) modal matrix.
  - c ) The eigen values ( **E** ) ( natural frequencies in diagonal matrix ) spectral matrix in ( rad/s ).

lam =

1.0e+009 \*

2.7285  
 0.8348  
 0.5110  
 0.4893  
 0.3307  
 0.2886  
 0.0632  
 0.0338  
 0.0016  
 0.0014  
 0.0003  
 0.0000  
 0.0000  
 0.0000  
 0.0000  
 0.0000

V =

Column 16

Columns 1 through 15

-0.0000	-0.0000	0.0023	-0.0000	-0.0036	0.0000	-0.0000	-0.0000	-0.2262	-0.1198	0.1022	-0.9602	-0.0254	0.0335	0.0172	0.0000
0.0000	0.0000	-0.0000	0.0000	0.9744	-0.0003	-0.0000	-0.0000	0.0678	0.2096	0.0057	-0.0450	-0.0013	0.0016	0.0008	-0.0000
0.0000	-0.0000	0.0000	-0.0000	-0.2249	-0.0001	0.0000	0.0000	0.2974	0.9100	0.0233	-0.1798	-0.0052	0.0065	0.0031	-0.0000
0.0000	0.0001	-0.0000	0.0000	0.0002	0.9999	0.0000	0.0000	0.0000	0.0001	0.0000	0.0005	-0.0138	0.0077	-0.0035	-0.0000
-0.0000	-0.0000	0.0000	-0.0000	-0.0000	-0.0162	-0.0000	-0.0000	-0.0001	-0.0005	-0.0001	0.0353	-0.8511	0.4757	-0.2188	-0.0000
-0.0000	-0.9999	-0.0000	0.0000	0.0000	0.0001	0.0000	0.0000	0.0000	0.0000	0.0000	-0.0000	0.0063	0.0082	-0.0067	-0.0000
-0.0000	0.0123	0.0000	-0.0000	-0.0000	-0.0000	-0.0000	-0.0000	-0.0000	-0.0000	-0.0000	-0.0001	0.5114	0.6640	-0.5455	-0.0000
0.0000	0.0000	-0.9729	0.0057	-0.0000	0.0000	0.0000	0.0000	0.2124	-0.0778	-0.0173	-0.0444	-0.0012	0.0016	0.0009	-0.0000
-0.0000	-0.0000	0.2312	-0.0003	0.0000	-0.0000	0.0000	0.0000	0.8962	-0.3261	-0.0740	-0.1773	-0.0046	0.0064	0.0035	-0.0000
-0.0000	0.0000	0.0054	0.9704	-0.0000	0.0000	0.0026	0.0007	0.0200	-0.0083	0.2394	0.0217	0.0004	-0.0015	-0.0014	-0.0000
0.0002	-0.0000	-0.0013	-0.2414	0.0000	-0.0000	0.0089	0.0024	0.0842	-0.0349	0.9622	0.0866	0.0016	-0.0060	-0.0058	-0.0000
-0.6912	0.0000	0.0000	0.0002	-0.0000	0.0000	-0.7152	-0.1030	0.0006	-0.0003	0.0070	0.0006	-0.0002	-0.0010	-0.0014	-0.0000
0.7226	0.0000	0.0000	0.0003	-0.0000	0.0000	-0.6838	-0.1007	0.0006	-0.0002	0.0064	0.0005	-0.0002	-0.0010	-0.0014	-0.0000
-0.0016	0.0000	-0.0000	-0.0000	0.0000	-0.0000	0.1441	-0.9889	0.0001	-0.0000	0.0010	-0.0010	-0.0041	-0.0203	-0.0286	-0.0000
0.0000	-0.0000	0.0000	0.0000	-0.0000	0.0000	-0.0019	0.0250	-0.0001	0.0000	-0.0038	-0.0227	-0.0814	-0.4069	-0.5716	-0.7071
0.0000	-0.0000	0.0000	0.0000	-0.0000	0.0000	-0.0019	0.0250	-0.0001	0.0000	-0.0038	-0.0227	-0.0814	-0.4069	-0.5716	0.7071

E =

1.0e+009 \*

Columns 1 through 15

Column 16

2.7285	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0.8348	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0.5110	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0.4893	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0.3307	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0.2886	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0.0632	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0.0338	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0.0016	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0.0014	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0.0003	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0.0000	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0.0000	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0.0000	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0.0000	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0.0000

- 2) The plotting of mode shapes against the distances between the mass moments of inertia of the system .
  - a) Before any modification changed of the stiffness of the shafts between the mass moments of inertia .

Corresponding Mode Shapes Before Modification

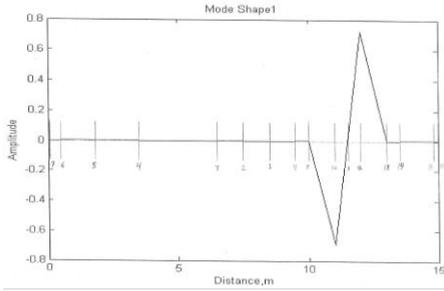


Figure 4: Mode Shape 1a

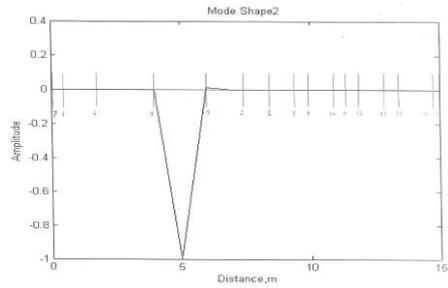


Figure 5: Mode Shape 2a

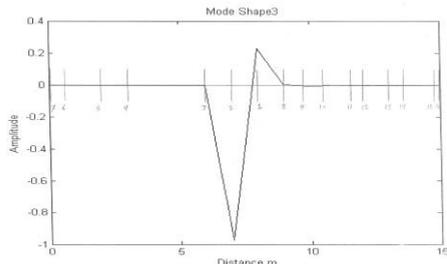


Figure 6: Mode Shape 3a

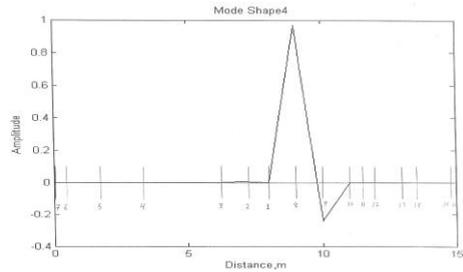


Figure 7: Mode Shape 4a

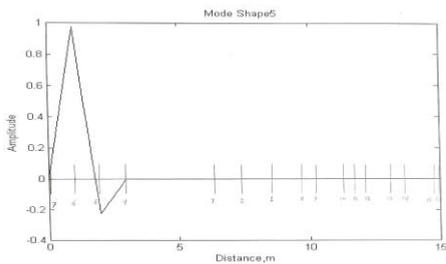


Figure 8: Mode Shape 5a

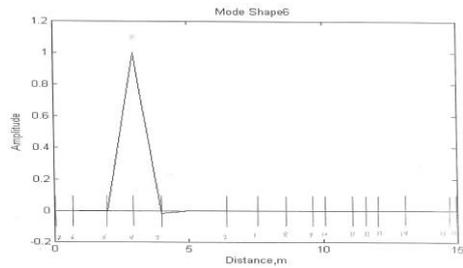


Figure 9: Mode Shape 6a

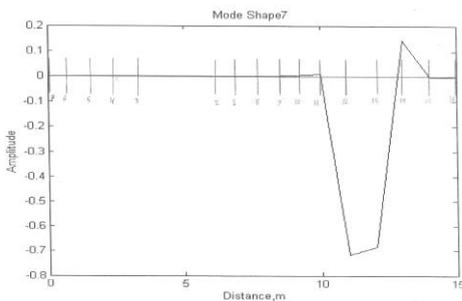


Figure 10: Mode Shape 7a

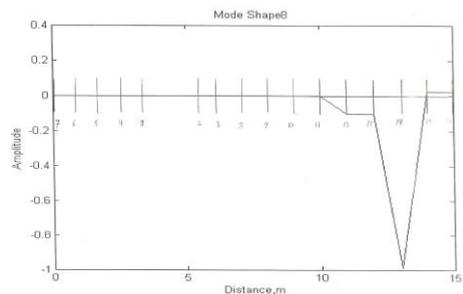


Figure 11: Mode Shape 8a

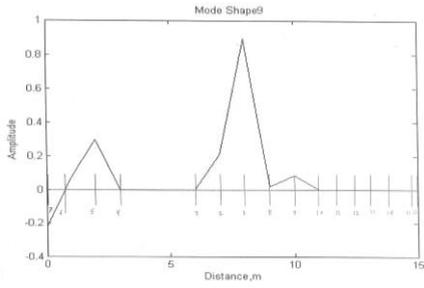


Figure 12: Mode Shape 9a

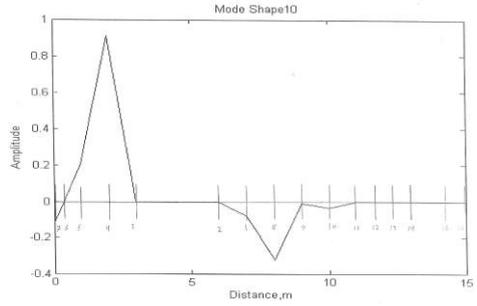


Figure 13: Mode Shape 10a

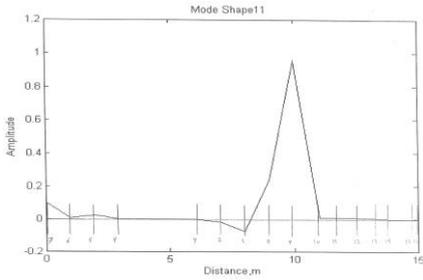


Figure 14: Mode Shape 11a

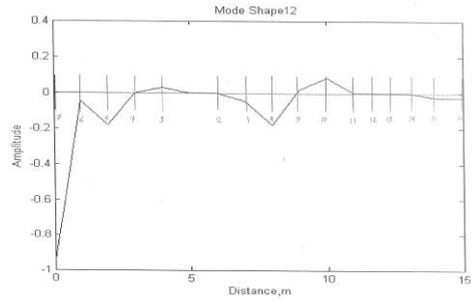


Figure 15: Mode Shape 12a

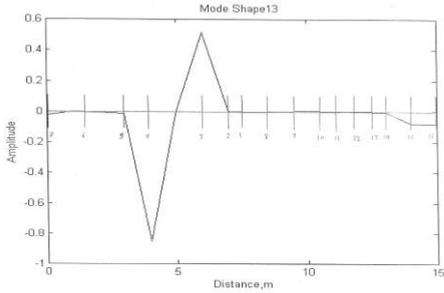


Figure 16: Mode Shape 13a

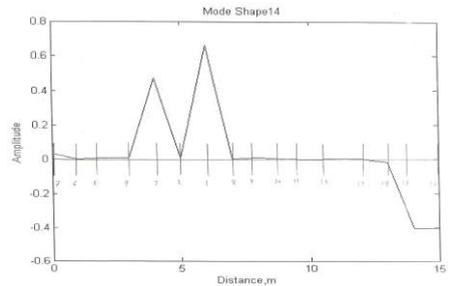


Figure 17: Mode Shape 14a

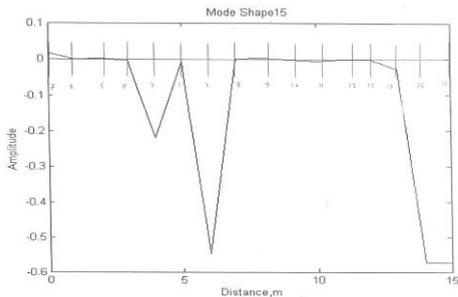


Figure 18: Mode Shape 15a

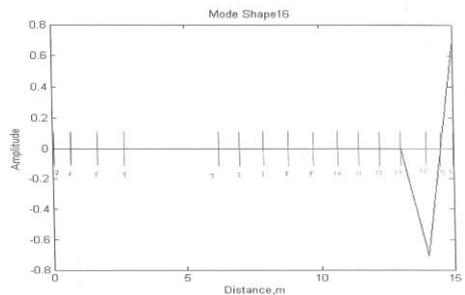


Figure 19: Mode Shape 16a

b) After modification changed of stiffnesses of the shafts between the mass moments of inertia.

**MODIFICATION TO DETERMINE THE SUITABLE DESIGN :**

When we changed all shafts of the system for propose of design modification in order to select optimum stiffness of shaft, it has been noted that no notable change in amplitude along the entire system i.e. shafts design is accurate, except change in stiffness of the shaft (  $K_3$  ) which is located between masses number ( 3 & 4 ). The shaft (  $K_3$  ) changed by addition ten percent and subtraction ten percent from value of its stiffness and can be seen that changed in the figures (graphs) below.

**Corresponding Mode Shapes After Modification**

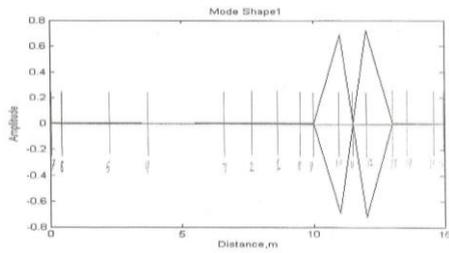


Figure 20: Mode Shape 1b

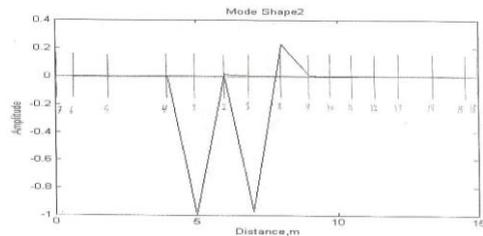


Figure 21: Mode Shape 2b

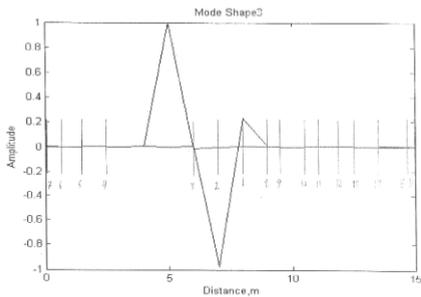


Figure 22: Mode Shape 3b

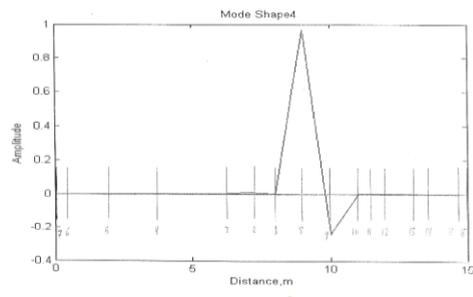


Figure 23: Mode Shape 4b

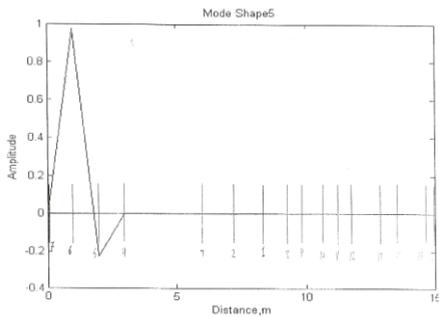


Figure 24: Mode Shape 5b

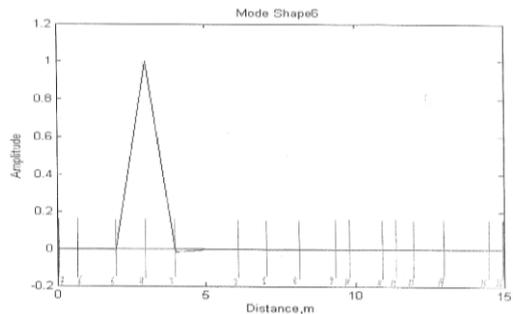


Figure 25: Mode Shape 6b

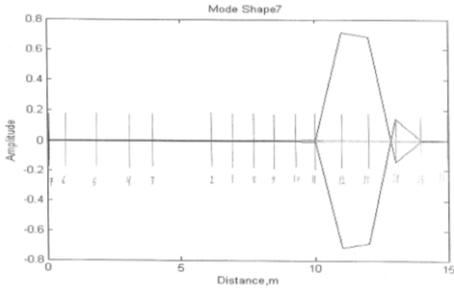


Figure 26: Mode Shape 7b

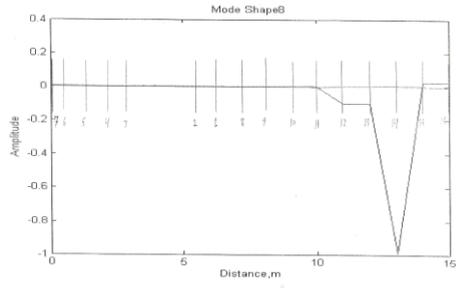


Figure 27: Mode Shape 8b

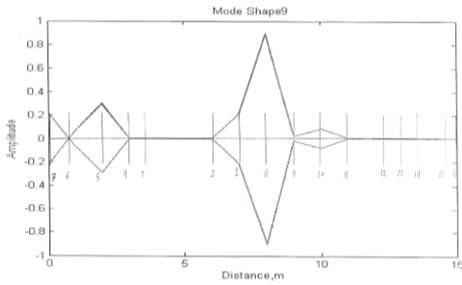


Figure 28: Mode Shape 9b

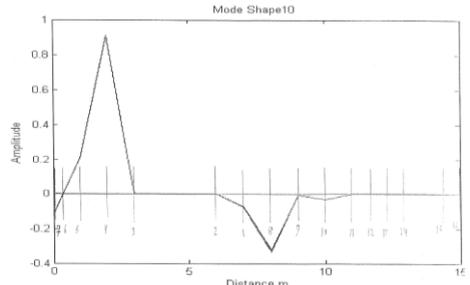


Figure 29: Mode Shape 10b

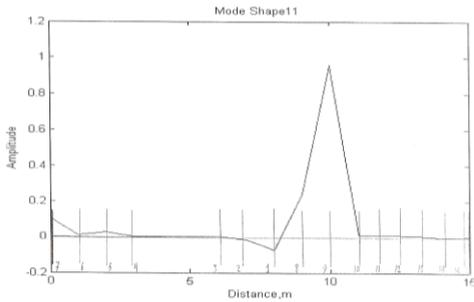


Figure 30: Mode Shape 11b

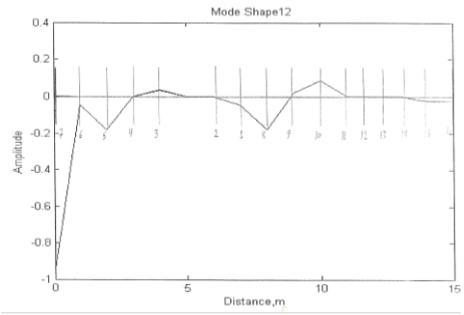


Figure 31: Mode Shape 12b

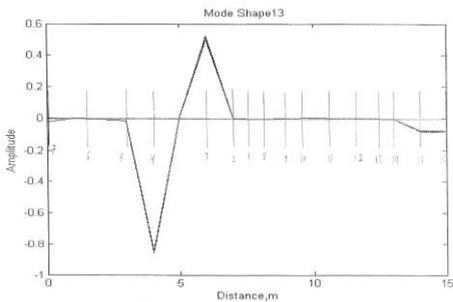


Figure 32: Mode Shape 13b

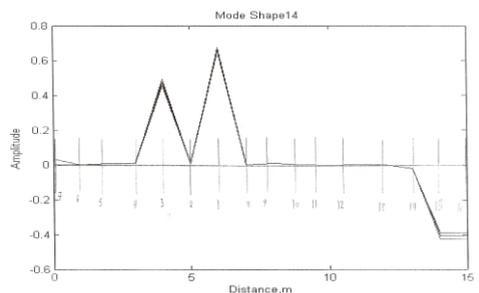


Figure 33: Mode Shape 14b

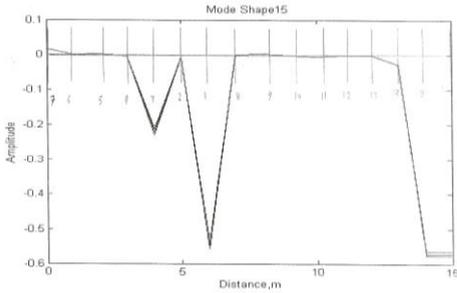


Figure 34: Mode Shape 15b

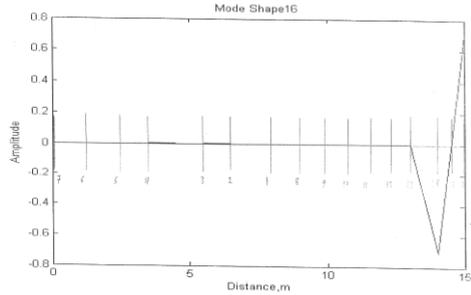


Figure 35: Mode Shape 16b

### 1. Desiccation

The mathematical model, Equation ( 1 ) of the mechanical model is shown in Figure ( 3 b ) are solved by Jacobi method using personal computer with Math-Lap program [10]. The solution of this model was done in two ways:

1. Considering the model without influence of modification of stiffnesses of shafts and plotting the results in the figures ( 4 to 19 ) where these figures are presented the relationship between the different distances of the shafts of the system and their responses amplitudes.

These relationships are expressed the torsional torques in each shaft of the system, that is expressed the torque amplification factors in each shaft of the system, whereas this torque amplification factor is the main source to failure the system especially when it is bigger, that is out of the operation condition of the system.

2. Considering the model with influence of modification of stiffnesses of shafts by changing each shaft in turns by increasing and decreasing its stiffnesses by ten percent of its original values and plotting the results in the figures ( 20 to 35 ).

By comparison the results from the figures before modification, figures ( 4 to 19 ), to the results from the figures after modification, figures ( 20 to 35 ), we have got that the results are same except in the shaft (  $K_3$  ). Therefore this shaft must remove from the system by another suitable one to avoid the failure of the system completely.

### 2. Conclusion of the Results :

For the solution of the problem one first of all define the problem estimate the physical system and then the mechanical system to ease the system to analyze in form of mathematical model for aim of computations and results.

The mathematical model, a powerful tool, has proved valuable in predicting the overall dynamic behavior of drive systems of rotary winding machines. Figure ( 3-b ) shows an arrangement of spring mass system of the drive system of the rotary winding machines.

Stated differential Equations (1) of a torsional drive system form the base of compilation of a program of numerical solution on computer. After allocation of individual constants one can numerically solve the transient states of regulating loop for the change of stiffnesses. By means of these calculations it is possible to optimize variable parameters of

regulating loop and to set in this manner the optimum regime of the whole torsional system of the rotary winding machine drive.

We have chosen Newton's second law method [2] for the theoretical analysis of vibration of the system for assemblies of equations of the torsional systems, and Jacobi method [10] for solution of these equations. The program of numerical mathematics was used here in Math-Lab programming language[9]. The calculation was done by means of a PC computer.

There were analyzed the responses of the drive of the system without and with change of characteristics of the shafts. At the same time all the results of the solution were compared. The useful result for the design procedure is obtained from the plotting of the relative amplitude values on the mass-elastic system diagram shown in the figures ( 20 to 35 ).

These plotted points join with a line ( this line is called the normal elastic curve ) Where this line crosses the line representing shaft torsional stiffness ( really it represents torsional flexibility as it is drawn to a length between masses inversely proportional to torsional stiffness ), this is a nodal point ( point of zero torsional amplitude ).

The shaft portion which has the nodal point is the shaft which have the largest vibratory vibration torque for that mode of vibration. For each natural frequency the number of nodes equal the mode number.

The location of the nodal points indicates which shaft sections have the major effect upon the frequency of that particular mode of vibration.

From stated calculations it is obvious that the importance of the influence of changing of shafts stiffnesses may lead to incorrect results. It is clearly seen that the non respectation of this changing may be substantially influence the course of investigated magnitudes so in quantitative manner as also the character of behaviour of the system .

Hence for unambiguous reply to question at what conditions and how significantly comes to force the influence of changing stiffnesses of shafts to the dynamics of the system, more through experimental investigations are needed.

Following the simple design procedures presented here will remove much of the mystery in finding a solution to torque amplification factor problems. Improved operating and maintenance procedures and increased use of automated rotary winding machine systems will afford additional major improvements in this area.

Now we conclude that the methods of vibration control may be grouped into :-

1. Reduction at the source by balancing of the moving masses, balancing of magnetic forces and control of clearances.
2. Isolation of source and isolation of sensitive equipment.
3. Reduction of the response by, alteration of natural frequency, energy dissipation and auxiliary mass.

The method and results of this work must be nevertheless considered only as a small step one of many leading to more detailed recognition of dynamic actions and events of changing of stiffnesses of shafts .

Also as to the problem solved in this work it is necessary to complete it by just a set of following tasks. So example, there will be necessary to solve:

1. The set of equations equation (1), with consideration of damped gears.
2. The system without reduction.
3. The system with consideration of effect of backlash in the gears on the system.
4. The system with consideration of the linear responses.

With the results in this work one can state that the used methods are generally suitable for investigation of the torsional vibrations of drives of other types of arrangements.

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## تحليل الاهتزازات الإلتوائية لماكينات اللف الدوارة

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### ملخص

اختيار هذا العمل الذي هو تحليل أنظمة ماكينات اللف الدوارة ( Analysis of Rotary Winding Machines System ) هو بسبب أن هذه الماكينات موجودة في البلد في مصنع الكيبلات الكهربائية في ضواحي مدينة عدن وهذا التحليل حقيقي وعملي من واقع المصنع وكل القيم والمقادير المستخدمة في هذا العمل أخذت كقيم فعلية لتلك الماكينات وبالتالي النتائج المتحصل عليها حقيقية ليست نتائج لمعطيات افتراضية.

إن التطبيق الجيد لنظام تصميم ماكينة اللف الدوارة يستدعي التحليل للنظام ليؤمن رد الفعل ( Reaction ) الناشئ عن تشغيل الماكينة بأن لا يتسبب عجز أو ضرر للماكينة وذلك من نشوء أو ظهور اهتزاز الماكينة، لأن اهتزاز الماكينة يؤدي إلى نتيجة تصنع أو تولد حمل ديناميكي كبير والذي غالباً يسبب العجز أو الضرر المفاجئ لأجزاء الماكينة أو تصنع نوعية إنتاج رديئة . يعتبر الاهتزاز ألتوائي ( Torsional Vibration ) ( الأوسع ظاهرة في أنواع الاهتزاز في النقل الميكانيكي . التوسع الدائم والمكثف للماكينات يستدعي دائماً زيادة طلب العمل من المخللين والمصممين وفي استخدام هذه الماكينات يعتبر قضيب المحرك ( Drive ) الأوسع تأثيراً على خاصية مجموعة القوى الفاعلة على أجزاء النظام مجتمعة .

الهدف من هذا العمل هو التحليل للاهتزازات الألتوائية ( Analysis of Torsional Vibrations ) لماكينات اللف الدوارة وتخفيض تلك الاهتزازات إلى الحد الأدنى المسموح به والذي بالتالي يؤدي إلى تخفيف مشاكل تكاليف الصيانة المتكررة . لتخفيض عزوم ديناميكية النظام ( System Dynamic Torques ) أنه من الضروري نشر أي بسط الترددات الطبيعية الألتوائية ( Spread Torsional Natural Frequencies ) للنظام . هذا يكون الأفضل إنجازه بتخفيض التردد الطبيعي ألتوائي الأساسي ( Fundamental Torsional Natural Frequency ) وذلك بتخفيض الصلابة الألتوائية ( Torsional Stiffness ) في موقع محور دوران ( Lead Spindle Location ) محركات ماكينة اللف الدوارة أو بصنع مرونة ( Flexibility ) أكبر لمحور الدوران ألتوائي وذلك بزيادة فصل عمود الإدارة ( Shaft Separation ) أو إذا من الضروري باستخدام المبادعة المرنة ( Flexible Spacer ) الألتوائية ، لأن الصلابة ( Stiffness ) الألتوائية تتناسب عكسياً مع طول عمود الإدارة ( Shaft Length ) .

لتحليل وحل الاهتزاز ألتوائي لنظام ماكينة اللف الدوارة يجب أولاً وقبل كل شيء تقدير معاملات ( Parameters ) النظام الفيزيائي مع الأخذ بعين الاعتبار المجموعة الحقيقية ( The Real Set ) لأجزاء ( Components ) ماكينة اللف الدوارة وتحويل ذلك إلى مودل ميكانيكي ( Mechanical Model ) . المودل الميكانيكي إلى مدى أبعد يعرف بمجموعة معادلات تفاضلية ( Differential Equations ) والتي تشكل المودل الرياضي ( Mathematical Model ) لنظام ماكينات اللف الدوارة . مجموعة المعادلات التفاضلية ( معادلة (1) في البحث ) هي عبارة عن معادلات الحركة ( Equations of Motion ) لكامل النظام المهتز وقد تم إيجادها باستخدام قانون نيوتن الثاني، وحل هذه المعادلات تم باستخدام طريقة العالم يعقوبي ( Jacobi Method ) لأنها طريقة سهلة للبرمجة وتعطي نتائج مضبوطة . البرمجة التي استخدمت في هذا العمل برمجة مثلاب . ( MathLab Program ) بسبب خاصيتها لحل مسائل الاهتزاز . أخيراً ومن واقع الحل والنتائج لهذا البحث فإنه يعطي رؤية جيدة لتندر إخفاق الماكينة وذلك بسبب الأخطاء في التصميم أو في حالات التشغيل.