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Numerical Solutions of Stiff and Nonstiff Ordinary Differential Equations Using Quadratic Spline Method

Abdulhabib A.A. Murshed

Department of Mathematics, Faculty of Applied Science, University of Thamar, Yemen, e-mail: habib20100@yahoo.com

ABSTRACT

This paper deals with the numerical solution of initial value problems(IVPs), for stiff and nonstiff ordinary differential equations by Quadratic Spline(QS) method. We convert a stiff and nonstiff equations to volterra integral equations and reduce a second order IVPs to a system of Volterra integral equations of the second kind . The numerical results shown to verify the conclusions .A comparison of the results generated from the QS formula was carried out with some existing Runge-Kutta methods of variety of means including the Geometric Mean method, the Contraharmonic Mean method, the Centroidal Mean method, the Harmonic Mean method, the Heronian Mean method, the Root Mean Square method, and the Arithmetic Mean formula and were found to compare favourably well. Good numerical results were obtained from the test examples and we conclude with numerical examples to justify the effectiveness of the QS method.
Key Words: stiff and nonstiff ordinary differential equation; quadratic spline method; volterra integral equations of the second kind..

1. INTRODUCTION

The quadratic spline Q(x) has continuous first derivatives at the knots $t_1 < t_2 < \ldots < t_n$.

The objective in quadratic spline is to derive a second order polynomial for each interval between data points(Chapra and Canale,1988, DeBoor,20011)

$$Q(x) = \begin{cases} Q_0(x) & t_0 \le x \le t_1 \\ Q_1(x) & t_1 \le x \le t_2 \\ . & \\ . & \\ . & \\ Q_{n-1}(x) & t_{n-1} \le x \le t_n \end{cases} .$$

Where $Q_i(x) = \dfrac{Q'_{i+1} - Q'_i}{2(t_{i+1} - t_i)}(x - t_i)^2 + Q_i'(x - t_i) + y_i .$ …(1.1)

Which is continuously differentiable on the entire interval $[t_1, t_n]$.

There are three conditions which define the function Q(x) uniquely on $[t_i, t_{i+1}]$ as given in equation(1.1), they are:

1- $Q_i(t_i) = y_i$

2- $Q'_i(t_i) = Q'_i$

3- $Q'_i(t_{i+1}) = Q'_{i+1}$

Now, in order for the quadratic spline function Q(x) to be continuous and to interpolate the table of data, it is necessary and sufficient that $Q_i(t_{i+1}) = y_{i+1}$ for i=1,2,…,n-1 in equation(1.1) with Q_1' arbitrary.

The result is:

$$Q_i(t_{i+1}) = \frac{Q'_{i+1} - Q'_i}{2(t_{i+1} - t_i)}(t_{i+1} - t_i)^2 + Q_i'(t_{i+1} - t_i) + y_i .$$

So, we get:

$$Q'_{i+1} = -Q'_i . + 2\left(\frac{y_{i+1} - y_i}{t_{i+1} - t_i}\right) \quad , \quad 1 \le i \le n-1 \qquad \text{… (1.2)}$$

This equation can be used to obtain the vector $[Q'_1, Q'_2, \ldots, Q'_n]^T$ starting with an arbitrary value for Q_1' (Fogiel,1983).

This quadratic spline will be used to find the solutions of volterra integral equation of the second kind as follows:

$$Q\ (x) = A_i(x)\ Q_i + B_i(x)\ Q_{i+1} + C_i(x)\ Q_i' . \qquad \text{… (1.3)}$$

Where

$$A_i(x) = 1 - \left(\frac{x - t_i}{h}\right)^2 \quad ,$$

$$B_i(x) = 1 - A_i(x) \quad ,$$

$$C_i(x) = \frac{1}{h}(x - t_i)\ (x - t_{i+1}) \quad , \quad h = t_{i+1} - t_i$$

For the continuity of $Q^{'}$ we require that:

$$Q^{'}_{i+1} = -Q^{'}_{i}. + \frac{2}{h}\left(y_{i+1} - y_i\right) \quad , \quad i = 1,2,....$$

Suppose now that $Q_0, Q_1, ..., Q_{r-1},\ Q^{'}_0, Q^{'}_1, ..., Q^{'}_{r-1}$ are known, then we put (1.3) in (1.1) we get:

$$Q_r = g_r + [\sum_{j=0}^{r-2} \int_{x_i}^{x_{i+1}} K(x_r, y, A_j(y)\ Q_j + B_j(y)\ Q_{j+1} + C_j(y)\ Q_j^{'}.]dy] +$$
$$\int_{x_{r-1}}^{x_r} K(x_r, y, A_{r-1}(y)\ Q_{r-1} + B_{r-1}(y)\ Q_r + C_r(y)\ Q_{r-1}^{'}.]dy \qquad (1.4)$$

This equation will be solved by iterations, and the integrals in (1.4) are replaced by Simpson's 1/3 rule (Delves and Walsh,1988).

2. CONVERTING INITIAL VALUE PROBLEMS TO VOLTERRA INTEGRAL EQUATIONS

***Example 1*)** To convert the following IVP to volterra integral equation:

$$\frac{dy}{dx} = f(x, y) \quad , \quad y(x_0) = y_0 \qquad (2.1)$$

We integrate (2.1) w.r.t. x from x_0 to x , we write:

$$y(x) - y(x_0) = \int_{x_0}^{x} f(t, y(t))dt$$

$$\therefore\ y(x) = y_0 + \int_{x_0}^{x} f(t, y(t))dt$$

which is a volterra integral equation of the second kind.

Example 2) To reduce the initial value problem:

$$\frac{d^2y}{dx^2} + y = \cos \ x \qquad (2.2)$$

$$, \ y(0) = 0 \ , \qquad (2.3)$$

$$y^{'}(0) = 0 \qquad (2.4)$$

to volterra equation , we let $u(x) = \frac{d^2y}{dx^2}$ then integrate one to have:

$$\frac{dy}{dx} = \int_0^x u(t)dt + c_1$$

With c_1 =0 after using the initial condition (2.3). We integrate this result again, using integration by parts :

$$y(x) = \int_0^x \int_0^s u(t)dtds$$

Let $u(s) = \int_0^s u(t)dt$ and *dv = ds*

So, *du = u(t)* and *v = s*

$$y(x) = \int_0^x \int_0^s u(t)dtds$$

$$= \left[s\int_0^s u(t)dt \right]_0^x - \int_0^x s \ u(s)ds$$

$$= x\int_0^x u(t)dt - 0 - \int_0^x s \ u(s)ds$$

$$= \int_0^x (x-s) \ u(s)ds = \int_0^x (x-t) \ u(t)dt$$

we get:

$$y(x)=\int_0^x (x-t)\ u(t)dt + c_2$$

where we also have c_2 =0 from using the initial condition (2.4),

$$y(x)=\int_0^x (x-t)\ u(t)dt \qquad (2.5)$$

For the final result (2.5) to be an integral equation, we use (2.2) to have

$$y(x)=\cos\ x-\frac{d^2y}{dx^2}=\cos\ x-u(x)$$

to substitute for the y(x) term outside the integral for (2.5) to become a volterra integral equation of the second kind in $u(x)=\frac{d^2y}{dx^2}$

$$\cos\ x-u(x)=\int_0^x (x-t)\ u(t)dt$$

$$u(x)=\cos\ x-\int_0^x (x-t)\ u(t)dt$$

Example 3) To reduce the initial value problem :

$$\frac{d^2}{dt^2}y(t)+P_1(t)\frac{d}{dt}y(t)+P_2(t)y(t)=g(t) \ ,$$

$$y(a)=y_0 \qquad y'(a)=y_1$$

Let $y_1(t)=y(t)$, $y_2(t)=y'(t)$, we get:

$$y'_1(t)=y_2(t) \quad ,$$

$$y'_2(t)=g(t)-P_1(t)y_2(t)-P_2(t)y_1(t)$$

Integrate both sides over [a,x] we get:

$$y_1(x)=y_0+\int_a^x y_2(t)dt \quad ,$$

$$y_2(x)=y_1+\int_a^x g(t)dt-\int_a^x P_1(t)y_2(t)dt-\int_a^x P_2(t)y_1(t)dt \ .$$

For more details about the relationship between ordinary differential equations and integral equations and reduction of initial value problems to volterra integral equations see for example(Jerri,1985 ,and Wazwaz,1997) .

3. NUMERICAL RESULTS

In this section, we present numerical results when the problems(3.1-3.10):

(i) First order IVPs:

Problem 1 :

$$\frac{dy}{dx} = xy \quad , \quad y(0)=0 \quad , \quad 0 \le x \le 1 \quad , \tag{3.1}$$

Where the exact solution is y(x)= $e^{\frac{x^2}{2}}$,

Problem 2 :

$$y' = -xy \quad , \quad y(0)=1 \quad , \quad 0 \le x \le 1 \quad , \tag{3.2}$$

Where the exact solution is y(x)= $e^{-\frac{x^2}{2}}$,

Problem 3 :

$$y' = -y \quad , \quad y(0)=1 \quad , \quad 0 \le x \le 1 \quad , \tag{3.3}$$

Where the exact solution is y(x)= e^{-x} ,

Problem 4 :

$$\frac{dy}{dx} = x(y - x^2 + 2) \quad , \quad y(0)=1 \quad , \quad 0 \le x \le 1 \quad , \tag{3.4}$$

Where the exact solution is y(x)= $x^2 + e^{\frac{x^2}{2}}$,

(ii) Second order IVPs:

Problem 5 :

$$y'' - 2(\cos(x-y))y = \frac{2}{x^2}\sin x \quad , \quad y(0)=0 \quad , y'(0)= 0, \quad 0 \le x \le 1 \quad , \tag{3.5}$$

Where the exact solution is y(x)= x e^{x} ,

Problem 6 :

$$y'' - y = -\frac{1333}{2400} x \quad , \quad y(0)=0 \quad , y'(0)=1 \quad , \quad 0 \le x \le 1 \quad , \tag{3.6}$$

Where the exact solution is y(x)= x ,

Problem 7 :

$$y'' + e^{-x+y} = -\frac{1}{x} e^{-x+2y} \quad , \quad y(0)=0 \quad , y'(0)=-1 \quad , \quad 0 \le x \le 1 \quad , \tag{3.7}$$

Where the exact solution is y(x)= x ,

Problem 8 :

$$y'' + y = -\cos x \quad , \quad y(0)=0 \quad , y'(0)=-1 \quad , \quad 0 \le x \le 1 \quad , \qquad (3.8)$$

Where the exact solution is $y(x)=(-1-\frac{1}{2}x)\sin x$,

Problem 9 :

$$y'' - (\frac{1}{x-y}\sin(x-y))y = \frac{2}{x^2}\cos x \quad , y(0)=0 \quad , y'(0)=0 \quad , \quad 0 \le x \le 1 \quad , \qquad (3.9)$$

The exact solution is : $y(x)= 1$,

Problem 10 (Stiff system):

$$y_1' = y_2 \quad , \quad y_1(0)=1 \quad , \quad 0 \le x \le 1$$

$$y_2' = -1000\, y_1 - 1001\, y_2 \quad , y_2(0)=-1 \qquad (3.10)$$

Where the exact solution is $y(x)= e^{-x}$

are solved using the Quadratic Spline (QS) method and comparing the obtained results with some methods including the Geometric Mean method, the Contraharmonic Mean method, the Centroidal Mean method, the Harmonic Mean method, the Heronian Mean method, the Root Mean Square method, and the Arithmetic Mean formula.

Tables (3.1-3.4) show the errors obtained by quadratic Spline method(QS), the fourth order Geometric Mean(GM) (Sanugi and Evans,1988), the Contraharmonic Mean(CoM), the Centroidal Mean(CeM), the Harmonic Mean(HM), the Heronian Mean(HeM) (Ponalagusamy and Senthilkumar,2008), the fourth order Root- Mean-Square(RMs4) (Yaakub and Evans,1993), and the third order Root- Mean-Square(RMs3) formula (Evans and Yaakub,1993).

The error in the numerical solution using Quadratic Spline formula(QS) compared with the exact solution are shown in Tables (3.5-3.6) .While the error in the numerical solution using Quadratic Spline formulae(QS) compared with Arithmetic Mean(AM) in (Sanugi and Evans,1988) and the exact solution are shown in Table (3.7).

Table (3.1) :Errors by using the various fourth order formulae for solving(3.1), h=0.1

X	*Error (QS)*	*Error (GM))*	*Error (CoM)*	*Error (CeM)*	*Error (HM)*	*Error (HeM)*	*Error (RMS3)*	*Error (RMS4)*
0.10	1.26048E-05	9.76569E-04	1.11026E-03	1.89784E-03	4.61308E-04	3.25467E-04	2.40415E-03	4.80117E-04
0.20	8.44982E-05	1.13862E-03	1.41870E-03	3.61521E-03	1.77921E-03	3.79466E-04	4.23212E-03	6.32071E-04
0.30	2.61854E-04	1.25842E-03	1.63542E-03	5.46699E-03	3.34006E-03	4.19369E-04	6.13173E-03	7.38278E-04
0.40	4.80859E-04	1.37320E-03	1.83262E-03	7.57058E-03	5.16784E-03	4.57590E-04	8.25632E-03	8.34024E-04
0.50	6.86416E-04	1.49668E-03	2.03652E-03	1.00472E-02	7.34790E-03	4.98704E-04	1.07363E-02	9.32225E-04
0.60	8.32555E-04	1.63723E-03	2.26240E-03	1.30419E-02	1.00018E-02	5.45497E-04	1.37181E-02	1.04037E-03
0.70	8.51714E-04	1.80192E-03	2.52252E-03	1.67370E-02	1.32907E-02	6.00329E-04	1.73823E-02	1.16442E-03
0.80	6.76874E-04	1.99809E-03	2.82906E-03	2.13690E-02	1.74269E-02	6.65642E-04	2.19601E-02	1.31026E-03
0.90	2.05026E-04	2.23419E-03	3.19578E-03	2.72486E-02	2.26918E-02	7.44258E-04	2.77540E-02	1.48449E-03
1.00	6.85999E-04	2.52055E-03	3.63925E-03	3.47902E-02	2.94617E-02	8.39629E-04	3.51664E-02	1.69504E-03

Table (3.2) : Errors by using the various fourth order formulae for solving(3.2) , h=0.1

X	*Error (QS)*	*Error (GM)*	*Error (CoM)*	*Error (CeM)*	*Error (HM)*	*Error (HeM)*	*Error (RMS3)*	*Error (RMS4)*
0.10	1.23964E-05	8.99897E-03	1.11200E-03	1.87438E-03	4.44409E-04	3.00030E-03	2.38611E-03	1.04558E-02
0.20	8.21925E-05	3.86186E-02	1.37540E-03	3.39442E-03	1.64616E-03	1.28158E-02	4.01468E-03	4.03794E-02
0.30	2.38577E-04	8.78933E-02	1.49803E-03	4.72227E-03	2.83845E-03	2.88204E-02	5.38497E-03	8.98601E-02
0.40	3.89868E-04	1.56623E-01	1.54870E-03	5.81988E-03	3.89355E-03	5.03963E-02	6.49785E-03	1.58769E-01
0.50	4.64298E-04	2.44713E-01	1.55071E-03	6.64831E-03	4.74106E-03	7.67280E-02	7.32710E-03	2.47035E-01
0.60	3.86826E-04	3.52232E-01	1.51619E-03	7.18156E-03	5.33761E-03	1.06839E-01	7.85259E-03	3.54739E-01
0.70	1.14633E-04	4.79507E-01	1.45342E-03	7.41018E-03	5.66133E-03	1.39639E-01	8.06810E-03	4.82219E-01
0.80	3.99640E-04	6.27249E-01	1.36905E-03	7.34191E-03	5.70989E-03	1.73978E-01	7.98322E-03	6.30190E-01
0.90	1.14688E-03	7.96696E-01	1.26891E-03	7.00048E-03	5.49841E-03	2.08701E-01	7.62275E-03	7.99897E-01
1.00	2.14169E-03	9.89786E-01	1.15820E-03	6.42308E-03	5.05679E-03	2.42694E-01	7.02448E-03	9.93287E-01
L.S.E.=	6.65604E-06	9.79676E-01	1.34144E-06	4.12559E-05	2.55711E-05	5.89003E-02	4.93433E-05	9.86619E-01

Where L.S.E. is the least square error.

Table (3.3) :Errors by using the various fourth order formulae for solving(3.3), h=0.1

x	*Error (QS)*	*Error (GM)*	*Error (HeM)*	*Error (RMS4)*
0.10	2.4564656542 E-03	1.9032496497 E-01	6.3432471495 E-02	1.9032519311 E-01
0.20	3.2354527611 E-03	3.8064989207 E-01	1.1881582590 E-01	3.8065039177 E-01
0.30	2.3240925411 E-03	5.7269834478 E-01	1.6697990177 E-01	5.7269916566 E-01
0.40	1.4670367364 E-03	7.6819388591 E-01	2.0867354171 E-01	7.6819508457 E-01
0.50	9.1548633227 E-03	9.6887568598 E-01	2.4457236439 E-01	9.6887732690 E-01
0.60	4.4472418149 E-03	1.1765141316 E-00	2.7528579503 E-01	1.1765162881 E-00
0.70	1.4984566722 E-02	1.3929265755 E-00	3.0136342481 E-01	1.3929293308 E-00
0.80	4.3886188919 E-03	1.6199933683 E-00	3.2330076317 E-01	1.6199968169 E-00
0.90	2.0869026894 E-02	1.8596743170 E-00	3.4154444095 E-01	1.8596785659 E-00
1.00	8.6447411923 E-04	2.1140257129 E-00	3.5649691646 E-01	2.1140308831 E-00
L.S.E.=	8.0770627787 E-04	4.4691047147 E-00	1.2709005145 E-01	4.4691265746 E-00

Table (3.4) : Errors by using the various fourth order formulae for solving(3.4), h=0.1

x	*Error (QS)*	*Error (CoM)*	*Error (CeM)*	*Error (HeM)*	*Error (RMS3)*	*Error (RMS4)*
0.10	3.77304E-05	3.32107E-03	5.66540E-03	9.74355E-04	7.19442E-03	1.43585E-03
0.20	2.52985E-04	4.21510E-03	1.06677E-02	1.13101E-03	1.25502E-02	1.87626E-03
0.30	7.74907E-04	4.81149E-03	1.58488E-02	1.24162E-03	1.79123E-02	2.16814E-03
0.40	1.39042E-03	5.32444E-03	2.14400E-02	1.34297E-03	2.36246E-02	2.41614E-03
0.50	1.90559E-03	5.82867E-03	2.76547E-02	1.44813E-03	2.99346E-02	2.65704E-03
0.60	2.15159E-03	6.36435E-03	3.47342E-02	1.56449E-03	3.70966E-02	2.91049E-03
0.70	1.89800E-03	6.96052E-03	4.29706E-02	1.69775E-03	4.54079E-02	3.19049E-03
0.80	9.24293E-03	7.64326E-03	5.27304E-02	1.85360E-03	5.52351E-02	3.50942E-03
0.90	1.08199E-03	8.43975E-03	6.44823E-02	2.03795E-03	6.70437E-02	3.88005E-03
1.00	4.46551E-03	9.38098E-03	7.88363E-02	2.25797E-03	8.14363E-02	4.31679E-03
L.S.E.=	3.64280E-05	8.80028E-05	6.21516E-03	5.09842E-06	6.63186E-03	1.86347E-05

Table (3.5) : Error in the Quadratic Spline formula for solving (3.5), (3.6) , (3.7), h=0.1

X	***Problem(3.5)***		***Problem(3.6)***		***Problem(3.7)***	
	Exact	*Error*	*Exact*	*Error*	*Exact*	*Error*
0.10	1.1051709181E-01	4.0892668892E-04	1.0000000000E-01	1.6835016834E-04	1.0000000000E-01	1.6277911971E-04
0.20	2.4428055163E-01	3.1745627412E-02	2.0000000000E-01	1.3649860064E-03	2.0000000000E-01	3.2509953573E-03
0.30	4.0495764227E-01	5.3708781052E-03	3.0000000000E-01	1.0187040093E-03	3.0000000000E-01	1.8803306125E-03
0.40	5.9672987906E-01	5.9974701153E-03	4.0000000000E-01	7.1177997461E-04	4.0000000000E-01	4.0332157964E-03
0.50	8.2436063535E-01	6.2002966060E-03	5.0000000000E-01	3.1897561616E-03	5.0000000000E-01	1.1632268023E-02
0.60	1.0932712802E+00	7.0774082760E-03	6.0000000000E-01	7.5510633051E-03	6.0000000000E-01	1.2162892241E-02
0.70	1.4096268952E+00	1.6882995802E-02	7.0000000000E-01	1.3382929083E-02	7.0000000000E-01	2.2228621474E-02
0.80	1.7804327428E+00	3.0888552837E-02	8.0000000000E-01	2.1837427546E-02	8.0000000000E-01	1.9495365182E-02
0.90	2.2136428000E+00	5.1725040514E-02	9.0000000000E-01	3.3411198240E-02	9.0000000000E-01	3.3952977842E-02
1.00	2.7182818285E+00	7.6922128144E-02	1.0000000000E+00	4.9464869352E-02	1.0000000000E+00	2.5322625841E-02
L.S.E.=	1.0992933304E-02		4.2896865191E-03		2.9818645078E-03	

Table (3.6) : Error in the Quadratic Spline formula for solving (3.8), (3.9) , h=0.1

x	***Problem(3.8)***		***Problem(3.9)***	
	Exact	*Error*	*Exact*	*Error*
0.10	-1.0049832501E-02	1.6708702333E-07	1.0000000000E+00	4.1638904804E-06
0.20	-2.0198653360E-02	5.0110315897E-09	1.0000000000E+00	8.2533006207E-08
0.30	-3.0445432706E-02	8.2843359905E-07	1.0000000000E+00	4.1871317080E-08
0.40	-4.0789120870E-02	2.0031598069E-06	1.0000000000E+00	2.1628329705E-07
0.50	-5.1228648502E-02	3.5229601849E-06	1.0000000000E+00	4.3075488065E-08
0.60	-6.1762926674E-02	5.3901333104E-06	1.0000000000E+00	2.8943668440E-07
0.70	-7.2390846994E-02	7.6084206739E-06	1.0000000000E+00	1.2660893844E-07
0.80	-8.3111281728E-02	1.0179976471E-05	1.0000000000E+00	3.6098208511E-07
0.90	-9.3923083912E-02	1.3108402754E-05	1.0000000000E+00	2.0794959710E-07
1.00	-1.0482608748E-01	1.6395700982E-05	1.0000000000E+00	4.3092950364E-07
	L.S.E.= 6.4836089974E-10		1.7854237555E-11	

Table (3.7) : Error in the Quadratic Spline formula and the Arithmetic Mean formula for solving (3.10), h=0.1

x	*QS*		*AM*		*Exact Solution*	
	Error of y_1	*Error of* y_2	*Error of* y_1	*Error of* y_2	y_1	y_2
0.10	1.2672E-06	1.2672E-06	1.6258196410E-04	1.6258196138E-04	9.0483741804E-01	9.0483741804E-01
0.20	2.64E-07	2.64E-07	6.0258025769E-04	6.0258119356E-04	8.1873075308E-01	8.1873075308E-01
0.30	1.154E-06	1.154E-06	9.4427919339E-04	9.4415394506E-04	7.4081822068E-01	7.4081822068E-01
0.40	4.76E-07	4.76E-07	1.2162216217E-03	1.2338599645E-03	6.7032004603E-01	6.7032004603E-01
0.50	1.069E-06	1.069E-06	1.4254477474E-03	1.1173258627E-03	6.0653065971E-01	6.0653065971E-01
0.60	6.49E-07	6.49E-07	1.9536824184E-03	3.6665157673E-01	5.4881163609E-01	5.4881163609E-01
0.70	1.025E-06	1.025E-06	1.7907004613E+00	1.7891750594E+03	4.9658530379E-01	4.9658530379E-01
0.80	7.92E-07	7.92E-07	2.3614505812E+02	2.3614668880E+05	4.4932896412E-01	4.4932896412E-01
0.90	1.013E-06	1.013E-06	3.3784643524E+04	3.3784641829E+07	4.0656965974E-01	4.0656965974E-01
1.00	9.16E-07	9.16E-07	4.8220874845E+06	4.8220874862E+09	3.6787944117E-01	3.6787944117E-01
L.S.E . = 8.399503 E-12						

4. CONCLUSION

Several examples were applied for illustration and good results were achieved. Good results depend on the selecting of the initial value of Q and the initial value of the derivative.

From Table (3.1), it can be seen that the errors satisfying
QS < HeM < RMS4 < GM < CoM < HM < CeM < RMS3.

From Table (3.2), it can be seen that the errors satisfying
QS < HM < CoM < CeM < RMS3 < HeM < GM < RMS4.

From Table (3.3), it can be seen that the errors satisfying
QS < HeM < GM < RMS3.

From Table (3.4), it can be seen that the errors satisfying

QS < HeM < RMS4 < GM < CoM < CeM < RMS3.

Tables (3.5) and (3.6) show that the Quadratic Spline method performs better accuracy compared with the exact solution.

Table (3.7) noted that the Quadratic Spline method gives the smallest error compared to the Arithmetic Mean in (Sanugi and Evans,1988) formula. The results generated were of high accuracy and have minimal errors. From the above results, it will be observed that the QS method is appropriate for stiff and nonstiff problems.

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الحلول العددية للمعادلات التفاضلية
الاعتيادية الصلبة وغير الصلبة بإستخدام طريقة الشرائح التربيعية

عبدالحبيب عبدالله احمد مرشد

قسم الرياضيات-كلية العلوم التطبيقية، جامعة ذمار- اليمن، e-mail: habib20100@yahoo.com

ملخص

يتناول هذا البحث الحل العددي لمسائل القيم الابتدائية للمعادلات التفاضلية الاعتيادية الصلبة وغير الصلبة بإستخدام طريقة الشرائح التربيعية . حيث قمنا بتحويل المعادلات الصلبة وغير الصلبة إلى معادلات فولتيرا التكاملية، وكذلك اختزال مسائل القيم الابتدائية من الرتبة الثانية إلى نظام من إلى معادلات فولتيرا التكاملية من النوع الثاني. النتائج العددية التي تم الحصول عليها تؤكد الاستنتاجات. تم عمل مقارنة بين النتائج العددية التي تم الحصول عليها بطريقة الشرائح التربيعية مع نتائج بعض طرق رونج-كوتا الموجودة بمتوسطات متعددة متضمنة طريقة المتوسط الهندسي، وطريقة المتوسط التوافقي العكسي ، وطريقة متوسط مركز الشكل الهندسي ، وطريقة المتوسط التوافقي ، وطريقة المتوسط الهيروني ، وطريقة جذر متوسط المربعات ، وطريقة المتوسط الحسابي. ووجدنا أن المقارنة حسنة. وتم الحصول على نتائج عددية جيدة من الأمثلة المستخدمة وبذلك نستنتج ومن خلال الأمثلة العددية قوة تأثير طريقة الشرائح التربيعية.