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Floating Point Genetic Algorithm Approach for Optimizing Unstructured Meshes

Ahmed Abdo Ali

University of Aden, Yemen, Email: ahmed_acc@yahoo.com

ABSTRACT

This paper presents a method for optimizing unstructured triangular meshes using a floating-point genetic algorithm. A mesh generation algorithm based on a modified advancing front method and sets of heuristic rules are used to generate the initial non-smooth triangular meshes for complex shapes. The developed mesh is then smoothed using a floating-point genetic algorithm that is more flexible than the usual binary genetic algorithms, and can handle non-smooth regions containing several local extrema. Three approaches are used in selecting the fitness function to be optimized in the genetic algorithm, namely, the triangle aspect ratio, the maximum angle at each node of the triangular mesh, and a weighted linear combination of both functions. The genetic algorithm has been tested and validated for a number of test cases covering a wide range of complex geometry applications. The results have shown a high degree of improvement in the quality of the smoothed meshes and an ability to handle non-convex regions.
Key Words: Genetic algorithms, mesh generation, unstructured meshes.

1. INTRODUCTION

The development of sufficiently smooth unstructured surface meshes (grids) plays an important role in the geometry processing of real world objects. In most application the term unstructured meshes refer to triangular and tetrahedral meshes in two and three dimension respectively. A primary application of unstructured meshes concerns the geometric modeling of complex real world objects, soft tissue modeling, and multi-resolution representation of complex shapes. Additionally unstructured meshes play a pivotal role in the numerical simulation of many physical problems in solid mechanics, geo-mechanics, and fluid dynamics.

Mesh generation has a huge literature and there are excellent references on structured and unstructured mesh generation [1-5], most grid generation techniques currently in use can fit into one of the three basic methods; Delaunay [6], Quadtree [7-8], and Advancing front method [5].

1.1 Grid optimization and smoothing

It is rare that any grid generation algorithm will be able to define a grid that is optimal without some form of post-processing to improve the overall quality of the grid. Most smoothing procedures [9-13] involve some form of iterative process that repositions individual vertices to improve the local quality of triangular grids.

The often used quality criteria for triangular grids are small aspect ratio and no angles very close to 0 or to 180. However, there is a variety of other optimization problems such as maximizing the minimum angle, minimizing the maximum angle, minimizing the maximum aspect ratio, minimizing the maximum circumradius and the radius of the containing circle of the incident triangles. Smoothing, averaging, and optimization-based methods are also used to improve the quality of triangular grids. Grid smoothing adjusts the locations of grid vertices in order to improve element shapes and overall grid quality. In grid smoothing, the topology of the grid remains invariant.

Laplacian smoothing is the most commonly used smoothing technique. Laplacian[14-16] smoothing is computationally inexpensive and fairly effective, but it does not guarantee improvement in grid quality. Similar to Laplacian smoothing, there are a variety of other averaging/smoothing techniques, which iteratively reposition nodes based on a weighted average of the geometric properties of the surrounding grid points (nodes). Canann [11] provides an overview of some of the common methods in use.

Optimization-based smoothing techniques measure the quality of the surrounding elements to a node and attempt to optimize by computing then local gradient of the element quality with respect to the node location. The grid vertices are moved so as to minimize a given distortion metric.

Some of the developments in this area include [11,12]. Parthasrathy [13]developed an optimization-based technique by solving a nonlinear, constrained, global optimization problem with the aspect ratio being the objective function to be minimized. Canann [11] presents optimization-based smoothing algorithm and recommends a combined Laplacian/optimization-based approach. An approach developed by Freitag [16] work to maximize the minimum angle in triangular grids by using an analogue of the steepest decent method for smooth functions. Amenta [9] presents theoretical results showing how some local triangle shape optimization be solved using generalized linear programming. Other efficient algorithms are presented and many distortion metrics are discussed and various optimization techniques are compared. Other optimization based methods include the works of [13] that is based on making use of posteriori error estimates, and [14] that

is based on the use of distortion metrics. The selection of good distortion metrics is also discussed in [15].

Recently, methods based on artificial intelligence concept have been used in grid generation and optimization. In [17] Neural Networks concepts have been used in finite element grid generations. Holder [18] presents a binary genetic algorithm for smoothing grids used in finite element analysis. A distortion metric is used to quantify the goodness of quadrilateral grid elements and serves as the fitness function for the genetic algorithm.

The objective of the paper is to:

- Develop an unstructured two-dimensional grid generation algorithm capable of triangulating complex geometrical shapes.
- Develop and apply a genetic algorithm for smoothing and optimization of triangular grids.

2. PROPOSED METHOD

The present method consists of a mesh generation algorithm based on the advancing front method for the generation of the initial grid to be smoothed, and a genetic algorithm to smooth the generated triangular mesh.

2.1 Grid Generation Algorithm

The algorithm is based on the advancing front method and a number of heuristic rules to ensure proper triangulation of arbitrary and simply connected regions into quality triangles. Several tests have been included into the algorithm to handle overlapping, edge crossing, and degenerate edges that may occur during the grid generation process. An efficient algorithm is also developed to generate the grid connectivity information needed during the grid generation process.

2. 2 Grid Smoothing

The developed grids normally contain poorly shaped triangles that may affect the generated grid when used in any specific application. The developed smoothing process is affected in two steps; first a diagonal swapping algorithm is first implemented followed by a floating-point genetic algorithm.

2. 2.1 Genetic Algorithms

The genetic algorithm starts from a set of chromosomes (assumed solutions) and evolves different but better sets of chromosomes (sets of solutions) over a sequence of iterations. In each generation (iteration) the fitness-measuring criterion (objective function) determines the suitability of each chromosome and, based on these values, some of them are selected for reproduction. The number of copies reproduced by an individual parent is expected to be directly proportional to its fitness value, thereby embodying the natural selection procedure, to some extent. The procedure thus selects more fit solutions; and less fit solutions are eliminated ,The structure of a simple genetic algorithm is shown in Figure 1.

```
Begin
t ← 0
Initialize P(t)
Evaluate P(t)
While (not termination-condition ) DO
Begin
t ← t + 1
select P(t) from P(t-1)
Crossover
Mutate
Evaluate P(t)
END
```

Figure 1: *Structure of a simple genetic algorithm.*

During iteration t, a genetic algorithm maintains a population of potential solutions (chromosomes, vectors)

$P(t) = \{x^t p \ldots, x^t n\}$.

Each solution is evaluated to give some measure of its "fitness", then, a new population (iteration $t+1$) is formed to establish new solutions. Crossover combines the feature of two parent chromosomes to form two similar offspring by swapping corresponding segments of the parents. Mutation arbitrary alters one or more genes of a selected chromosome, by a random change with a probability equal to the mutation rate.

The present genetic algorithm uses proportional selection, elitist model, one point crossover and uniform mutation. A geometrical procedure that will produce an object function for optimization works in the following manner.

2.2.2 Genetic Algorithm parameters

• *Population size*

Population size is one of the parameters affecting the algorithm's convergence. The larger, more diverse population takes longer to converge on a solution, but is more likely to find a solution because of its diverse gene pool. For every internal node in the grid, 50 generation of populations are generated and every population contains 100 chromosomes (nodes).

Creating a population of chromosomes initializes the process. Each chromosome contains two floating random numbers, the first for x-coordinate and the second for y-coordinate.

Each member of every chromosome is generated using random number generator.

• *Evaluation function*

For every random node (chromosome) generated the summation of areas for all triangles around that node are computed and it must be equal the summation of areas A for all triangles around that initial node N (to be moved to optimum location).The fitness function for every node is described later.

• *Selection Process*

For the selection process (selection of a new population with respect to the probability distribution based on fitness values), a roulette wheel with slots sized according to fitness is used.

A roulette wheel is constructed as follows:

1. Calculate the fitness value *fivalu(Ni)* for each chromosome(node *Ni*) (i=1,2,…, Pop_Size).
2. Find the total fitness of population (i) _
   1 Σ = = *pop size*
   i F fivalu N.
3. Calculate the probability of a selection *pi* for each chromosome
   (node *Ni*) (i=1,2,…, Pop_Size): *pi*
   = *fivalu(Ni)* /F
4. Calculate a cumulative probability *qi* for each chromosome
   (node *Ni*) (i=1,2,…, Pop_Size):
   $qi=\Sigma^{i}j=1$ Pj
5. Calculate a cumulative fitness
6. Finally select survivors using cumulative fitness probability and based on elitist model :

-Generate a random (float) number r between [0..1],for each chromosome i.e. Pop_Size times; each time(for every chromosome) if r < q1 then select the first chromosome (*N*1);

-Otherwise select the i-th chromosome *N*i (2 ≤ i ≤ Pop_Size) such that *qi-1 < r < qi.*

• *Crossover Probability:*

Crossover probability usually ranges from 0.01 to 1.0. Crossover reflects the likelihood that future population of nodes will contain a mix of information from the previous generation of nodes. A rate of 0.5 means that a child node will contain about 50% of its location from one parent node and the rest from the second parent node. A rate of 1.0 means that no crossover will occur, or only clones of the parents will be evaluated. A crossover probability of 0.8 is used in the current algorithm.

• *Mutation rate*

The mutation rate can vary from 0.0 to 1.0. The higher the mutation rate, the more likely future nodal chromosomes will contain some random values. Since mutation occurs after crossover, a mutation rate that is too high will prevent crossover from having much if any effect. A random uniform mutation with a probability of 0.15 is used in the present algorithm.

• *Evaluation (fitness) function*

Three approaches are used in selecting the function to be optimized in the genetic algorithm, namely, the least square error of the variation of angles, the average of the aspect ratio of triangles at each node of the triangular grid, and a linear combination of both approaches. In this method evaluation function is in a composite form, made up of the least square error of the angles and average of aspect ratio for the triangles. Numerical experimentation indicates that the third approach is the best criteria for the selection of the fitness function.

The fitness function is computed at every selected node. In every iteration, the absolute value of the change of the fitness function F and its maximum value is computed. The process is repeated for every iteration until convergence is achieved. A fixed value of 0.01 is set for the differences in the fitness function as criteria for convergence.

3. RESULTS

The optimization algorithm is implemented for complex geometrical shapes and the results are shown in Figures 2 through 9. The optimization algorithm selectively moves the nodes of the poorly shaped triangles based on the fitness function which results in a marked improvement in the grid quality. The results clearly show that the improvement in the triangles quality from the initial grid to the optimum.

Figures 7 through 9 compares the variation of the fitness function before and after optimization for nodes of the poorly shaped triangles. α= 0.8 and β =0.2. for the case of the cartoon-type rooster shape The curves of figure 8 show that the optimization using the genetic algorithm has resulted in improved values of the fitness function. The fitness values for most nodes ranges from 0.5 to 0.85, after smoothing the fitness values improved to a range of 0.85 to 0.94.

The analysis of the curves in figure 1-6 shows that the fitness values of the initial grid for most nodes are in range 0.5 to 0.85. But after optimization with diagonal swapping the range improved to a range of 0.88 to 0.97. The results shown in figures 7-9 show that smoothing using the genetic algorithm have made a marked enhancement of the triangles quality.

The triangles quality have improved and the range of the quality of the most triangles is improved from (0.11 – 0.92) to (0.91 – 97).

Figure 2: Initial and smoothed grids of cartoon-type rooster shape.

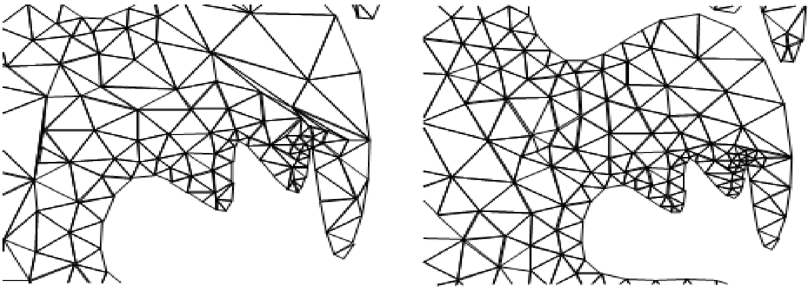

Figure 3: Enlarged region of initial and smoothed grids of cartoon-type rooster shape.

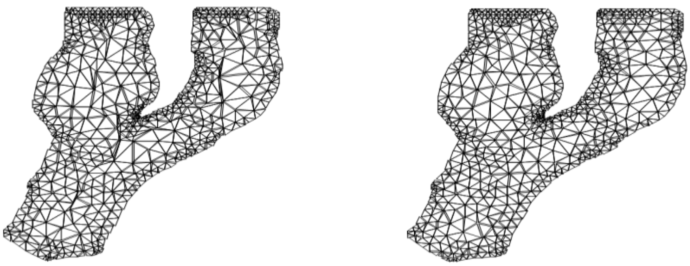

Figure 4: Initial and smoothed grids of an artificial heart model section

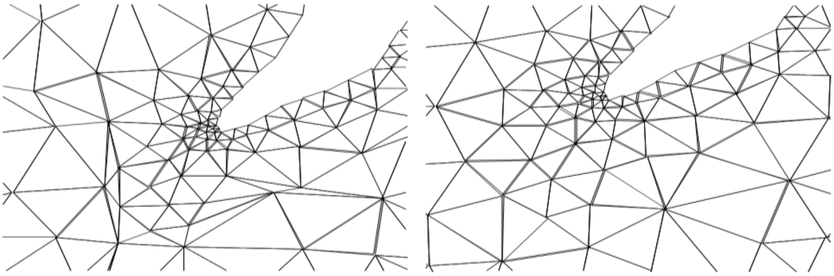

Figure 5: Enlarged region of Initial and optimum grid of an artificial heart section

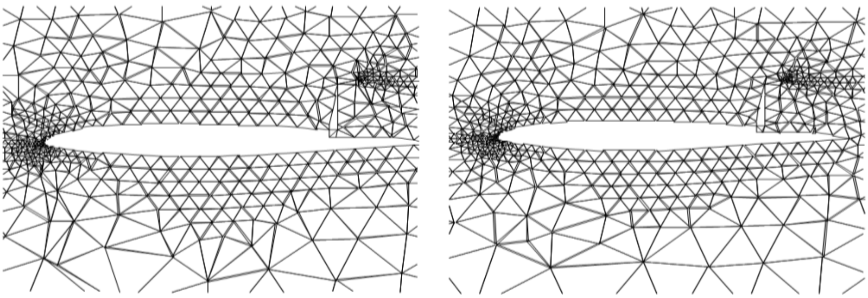

Figure 6: Enlarged region of Initial and optimum grid of an airfoil with a spoiler

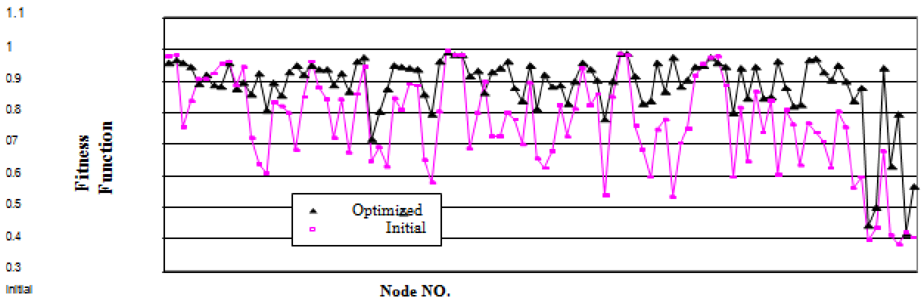

Figure 7: Curves of initial and optimum fitness function for cartoon-type rooster shape.

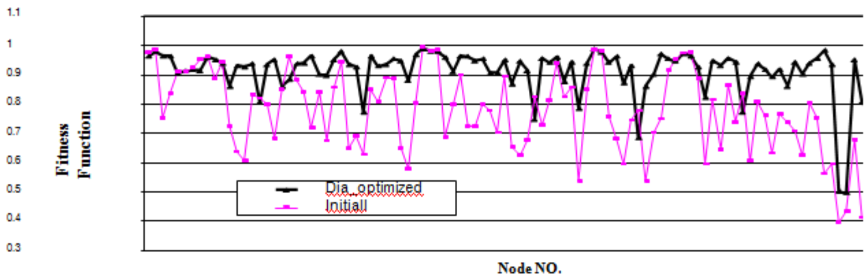

Figure 8: Curves of initial and optimum fitness function for cartoon-type rooster shape

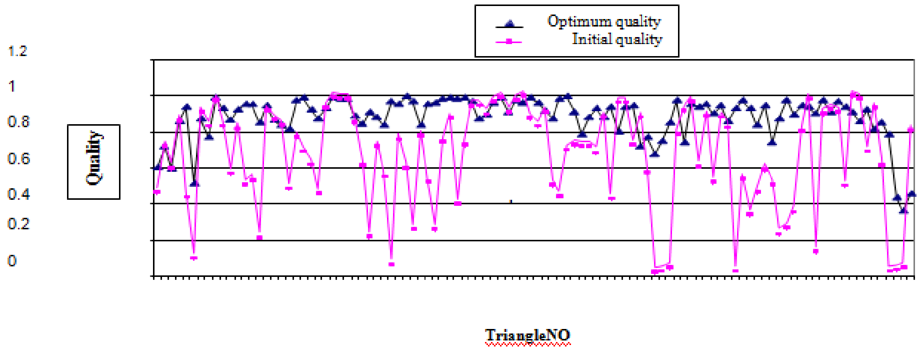

Figure 9: Curves of initial and optimum triangles quality for Cartoon-type rooster shape.

4. Conclusions

In this paper an unstructured triangular grid generation and smoothing algorithm is developed for arbitrary two-dimensional regions. The grid generation algorithm is based on a modified advancing front method and a set of heuristic rules to guarantee the efficient triangulation of complex regions. The developed algorithm gives a high priority for the smallest angle between any two adjacent segments during the triangulation process. The developed heuristics rules are designed to avoid many of the problems encountered during triangulation; such as, front overlapping and intersection, close proximity of neighboring triangles.

Additionally, the algorithm uses an efficient branching strategy for triangle construction and selection. The grid generation algorithm also includes an efficient fast permutation algorithm to establish the connectivity of the triangular grid that is needed for the advancing front algorithm and for post-processing the developed grids. The results show that the grid generation algorithm is capable of generating high quality triangles for any complex two dimensional region. A smoothing algorithm is then developed to optimize the quality of the developed grid. The optimization process is implemented in two steps.

First, diagonal swapping is implemented, followed by the floating-point genetic algorithm. Diagonal swapping is used according to a triangle quality that is based on the aspect ratio. The second step in the optimization process employs a floating-point genetic algorithm that is more flexible than the usual binary genetic algorithms. Three approaches are used in selecting the function to be optimized in the genetic algorithm, namely, the least square error of the variation of angles, the average of the aspect ratio of triangles at each node of the triangular grid, and a linear combination of both approaches. The results clearly show the marked improvement in the quality of the optimized grids and the ability of floating point genetic algorithm to handle non-convex regions.

Future research work may include the extension of the present method to the more difficult case of three dimensions to establish smooth tetrahedral grids for 3D regions.

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الخوارزمية الوراثية (الجينية)
لتحقيق الأمثلية (الوصول الأمثل)للشبكات الغير بنيوية

أحمد عبده علي

جامعة عدن/كلية التربية-عدن، Ahmed_acc@yahoo.com

ملخص

هذه الورقة تقدم طريقة للوصول الأمثل إلى الشبكات الثلاثية الغير بنيوية باستخدام خوارزمية النقطة العائمة الوراثية.
لقد تم استخدام خوارزمية لتوليد الشبكة الابتدائية الثلاثية الغير مستوية (ناعمة) بالاعتماد على طريقة الواجهة المتقدمة المعدلة وكذا مجموعات القوانين التجريبية.
الشبكة المتولدة تم تنعيمها باستخدام خوارزمية النقطة العائمة الوراثية التي تعتبر أكثر مرونة من الخوارزمية الثنائية الوراثية ، والتي تستطيع التعامل مع المناطق الغير مستوية (ناعمة) والتي تحتوي على عدة نهايات محلية (نتوءات محلية)
تم استخدام ثلاث طرق لاختيار الدالة الملائمة والتي سيتم تحقيق الأمثلية لها باستخدام الخوارزمية الوراثية وهذه الطرق هي:
نسبة أبعاد المثلث ، الزاوية العظمى لكل عقده من الشبكة الثلاثية ، ومركب خطي وموزون من الدالتين.
الخوارزمية الوراثية تم فحصها والتأكد من صلاحيتها باستخدام عدد من حالات الاختبار والتي تغطي مجموعة واسعة من تطبيقات الأشكال الهندسية المعقدة.
دلت النتائج على درجة عالية من التحسن في وجود الشبكة المنعمة والقدرة على التعامل مع مناطق النتوءات الغير محدبة.